

Electrostatics

M.M. - 16

(i) electric charge
and
fields(ii) electrostatic
potential

(iii) capacitance

Chapter - 1 :- Electric charge and fields

1:- Electric charge :- "charge is the property associated with matter due to which it produces and experience electrical and magnetic effects"

or

electric charge is an intrinsic property of elementary particles of matter which gives rise to electric force b/w various objects. it is denoted by 'q'

* Direction :- it is a scalar quantity and its direction is from positive terminal to negative terminal.

* Unit :- * S.I. unit is coulomb (C)

* C.G.S. unit is stat-coulomb

* e.m.u. (electro magnetic unit) is
ab - coulomb

$$1 \text{ coulomb} = 3 \times 10^9 \text{ stat-coulomb}$$

$$1 \text{ coulomb} = \frac{1}{10} \text{ ab-coulomb}$$

②

Dimensional formula -

$$\therefore \text{current } (I) = \frac{\text{charge } (Q)}{\text{Time } (t)}$$

$$\Rightarrow Q = It$$

$$Q = [AT]$$

Type of charges:-

There are Two types of charge

- * Positive charge
- * Negative charge

positive charge - When glass rod is rubbed with silk cloth The charge produce on glass rod due to loss of electrons is called positive charge.

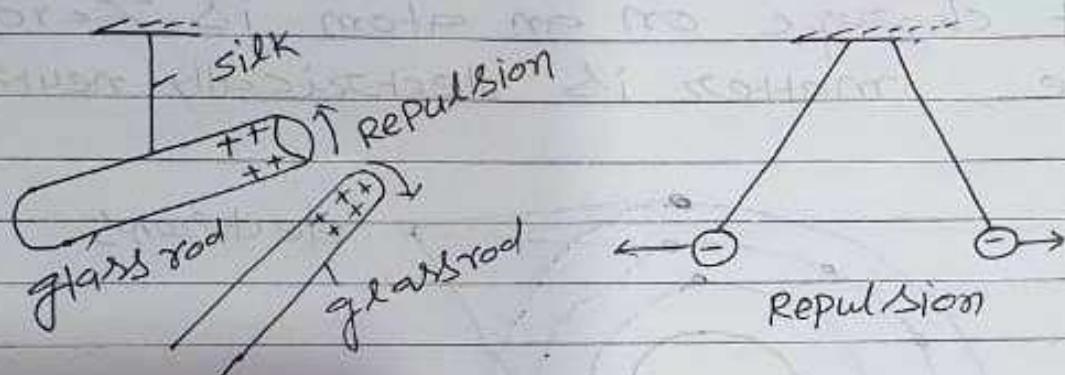
Negative charge - When ebonite rod is rubbed with woollen cloth or cat skin, The charge produce on ebonite rod due gain of electron is called negative charge

①

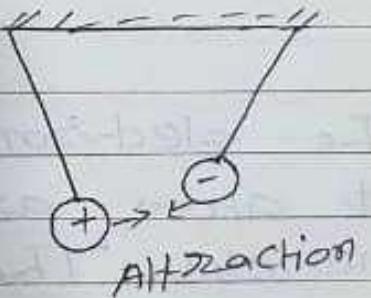
Note:- A proton has positive charge (+e) and electron has a negative charge (-e) where $e = 1.6 \times 10^{-19}$ coulomb

Nature of charges-

- * charges with same electrical sign repel each other.



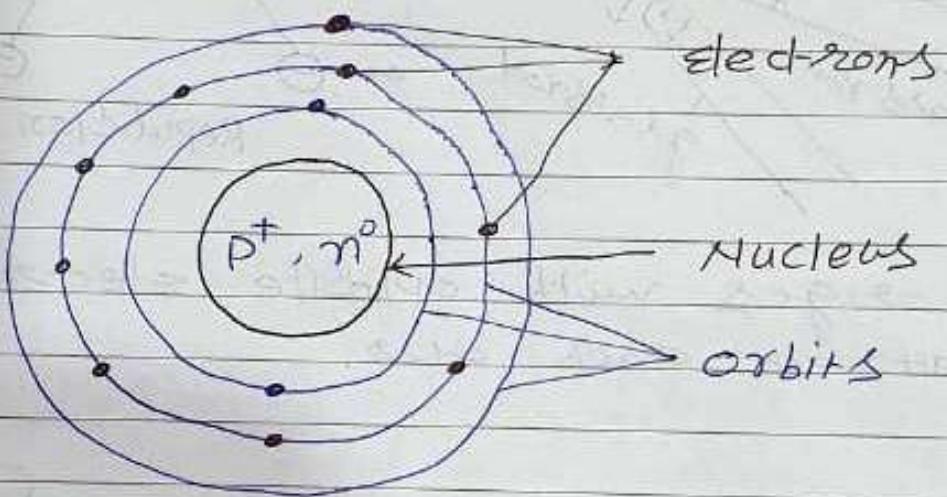
- * charges with opposite electrical sign (+ and -) attract each other.



W

Electronic Theory of electric charging:-

All matter is made up of atoms. An atom consists of a small central nucleus containing protons and neutrons, around which revolve a number of electrons. The amount of total positive charges and the amount of negative charges in an ~~an~~ atom are equal. Hence the net charge on an atom is zero. Therefore, matter is electrically neutral.



The electrons of the outer shell of an atom are loosely bound to the nucleus. The energy required to remove an electron from the surface of a material is called its 'work function'. When two bodies are rubbed together, electrons in the outermost orbits ~~with different~~ of atoms of a body get transferred to the other body.

(5)

The materials which lose electrons are said to have positive charge, whereas those gaining electrons are said to have negative charge.

(2)

The materials which lose electrons are said to have positive charge, whereas those gaining electrons are said to have negative charge.

conceptual problems:-

Q1- Is the mass of a body affected on charging?

Ans:- Yes, electrons have a definite mass. The mass of a body slightly increases if it gains electrons while the mass decreases if the body loses electrons. The mass decreases if the body loses electrons.

$$\text{mass transferred} = \text{No. of electron transferred} \times \text{mass of electron}$$

$$= n \times m_e$$

$$\text{where } m_e = 9.1 \times 10^{-31} \text{ kg}$$

⑥

Basic properties of electric charge:-

The basic properties of electric charges are -

- (1) Additivity of electric charges:- electric charge is additive i.e. total charge of a system is the algebraic sum of all the individual charges located at different points inside the system.

If a system contains charges q_1, q_2, \dots, q_n . Then its total charge is

$$q_{\text{Total}} = q_1 + q_2 + \dots + q_n$$

example:- The total charge of a system containing four charges 2NC , -3NC , $+4\text{NC}$ and -5NC is

$$q_{\text{Total}} = q_1 + q_2 + q_3 + q_4$$

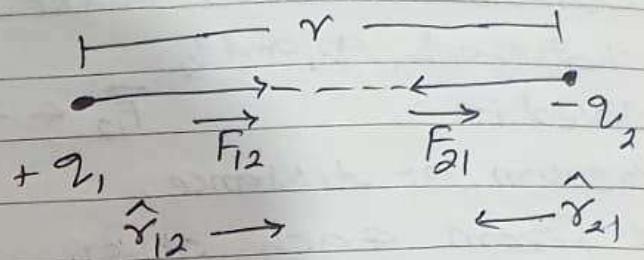
$$q_{\text{Total}} = 2 - 3 + 4 - 5$$

$$q_{\text{Total}} = -2\text{NC}$$

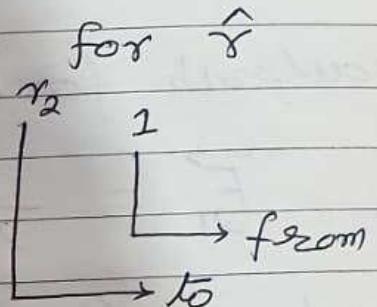
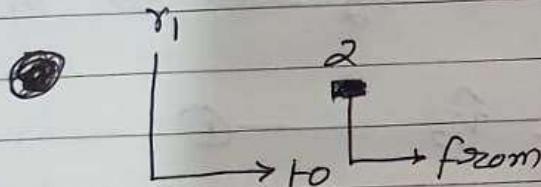
Numerical:- (NCERT) :- A system has two charges $q_A = 2.5 \times 10^{-7} \text{C}$ and $q_B = -2.5 \times 10^{-7} \text{C}$ located at A ($0, 0, -15\text{cm}$) and B ($0, 0, +15\text{cm}$) respectively. What is the total charge on the system?

on each other are equal in magnitude and opposite in direction.

Note:- ①



② Remember convention for $\hat{\gamma}$



(B)

② Invariance of electric charge:-

The magnitude of charge on a body does not vary with the speed of the body.

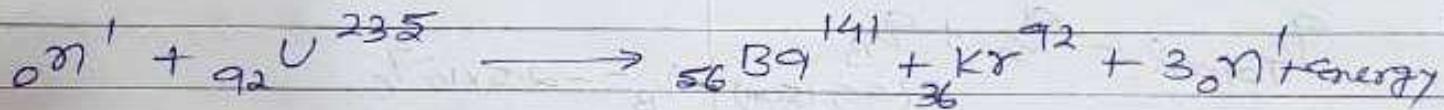
i.e.

$$q_{(\text{at rest})} = q_{(\text{in motion})}$$

③ conservation of electric charge:-

The net electric charge in an isolated system remains constant.

Examp- charge is conserved during the fission of a $^{92}_{\text{U}} \text{U}^{235}$ nucleus by a neutron.



Total charge before fission = $0 + 92 = 92$

Total charge after fission = $56 + 36 + 3 \times 0$
= 92

Hence total electric charge is conserved.

(9)

Quantization of electric charge:-

is THE PROPERTY of THE body by virtue of which any charged body can have charge which is an integral multiple of THE basic or ~~elect~~ elementary charge (e).

$$\text{i.e. } q = \pm ne$$

where

$$q = \text{Total charge}$$

$$n = 1, 2, 3, \dots$$

$$e = \text{elementary charge} = 1.6 \times 10^{-19} \text{ C}$$

$$\text{i.e. } q = \pm e, \pm 2e, \pm 3e, \dots \text{ etc.}$$

charge on a body can never be $\pm \frac{e}{2}, \pm \frac{3e}{2}, \pm \frac{5e}{2}, \dots$ ~~etc.~~
cause of quantization of electric charge!

electrons are always transferred in integral values from one object to another object.

Numerical exam:- How many electrons form one coulomb of charge?

Ans:-

$$q = 1 \text{ C}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$n = ?$$

$$\therefore q = ne \Rightarrow n = \frac{q}{e} = \frac{1}{1.6 \times 10^{-19}} = \frac{1 \times 10^19}{1.6}$$

$$q = \frac{10 \times 10^{18}}{1.6}$$

$$\therefore q = 6.25 \times 10^{18} \text{ electrons}$$

(10)

Numerical:-

A piece of polythene is rubbed with wool and it has been found to acquire a negative charge of $3 \times 10^{-7} C$

- (i) How many electrons are transferred from wool to ~~the~~ piece of polythene?
- (ii) Is there a transfer of mass from wool to polythene? If yes how much?

Ans:-

Given -

$$q = 3 \times 10^{-7} C$$

$$e = 1.6 \times 10^{-19} C$$

$$\therefore q = ne$$

$$n = \frac{q}{e}$$

$$n = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}} = 1.875 \times 10^{12}$$

- (ii) Yes, mass is transferred from wool to polythene

Mass transferred = $n \times$ mass of one electron

$$= 1.875 \times 10^{12} \times 9.1 \times 10^{-31}$$

$$= 1.706 \times 10^{-18} kg$$

Numerical:- Can an object have a charge of $2.8 \times 10^{-18} C$? Justify your answer

Ans:- $q = 2.8 \times 10^{-18} C$, $e = 1.6 \times 10^{-19} C$

$$q = ne \Rightarrow n = \frac{q}{e}$$

(11)

$$n = \frac{2.8 \times 10^{-18}}{1.6 \times 10^{-19}} = 17.5$$

which is not possible because as per quantization of electric charges, number of elementary charges must be an integer.

(NCERT)

Numerical: How much positive and negative charge is there in a cup of water?

Ans:- In a H₂O molecule, there are 10 protons and 10 electrons

$$\text{molecular mass of water} = 2 + 16 = 18 \text{ g}$$

$$\begin{aligned}\text{Number of molecules in } 18 \text{ g of water} \\ &= \text{Avogadro's Number} \\ &= 6.023 \times 10^{23}\end{aligned}$$

Suppose the mass of water contained in a cup is 250 gm

$$\begin{aligned}\text{Number of molecules in } 250 \text{ gm of water} \\ &= \frac{6.023 \times 10^{23} \times 250}{18} \\ &= 6.023 \times 10^{24}\end{aligned}$$

$$\begin{aligned}\text{No. of protons or electrons in } 250 \text{ gm of water} \\ &= 10 \times 6.023 \times 10^{24} \\ &= 6.023 \times 10^{25}\end{aligned}$$

Total charge in 250 gm of water

$$q = ne$$

$$q = 6.023 \times 10^{25} \times 1.6 \times 10^{-19} = 9.637 \times 10^6 \text{ C}$$

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Numerical! If a body gives out 10^9 electrons every second. How much time is required to get a total charge of 1C from it?

Ans:- No. of electrons given out by the body in one second (n) = 10^9

$$\begin{aligned} \text{charge given out by the body in} \\ \text{one second} &= ne \\ &= 10^9 \times 1.6 \times 10^{-19} \\ &= 1.6 \times 10^{-10} \text{ C} \end{aligned}$$

Time required to get a charge of 1.6×10^{-10} C = 1s

$$\begin{aligned} \text{Time required to get a charge of} \\ 1 \text{C} &= \frac{1}{1.6 \times 10^{-10}} \text{ s} = 6.25 \times 10^9 \text{ s} \end{aligned}$$

$$= \frac{6.25 \times 10^9}{365 \times 24 \times 3600} \text{ years} = 198.18 \text{ years}$$

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Numerical: A paisa coin is made up of Al-mg alloy and weight 0.75 gm. It has a square shape and its diagonal measure 17 mm, it is electrically neutral and contains equal amounts of positive and negative charges.

Ans:-

$$\text{mass of a paisa coin} = 0.75 \text{ g}$$

$$\text{Atomic mass of aluminium} = 26.9815 \text{ gm}$$

$$\text{Avogadro's number} = 6.023 \times 10^{23}$$

∴ Number of Al atoms in one paisa coin

$$n = \frac{6.023 \times 10^{23}}{26.9815} \times 0.75 = 1.6742 \times 10^{22}$$

As charge number of Al is 13, each atom of Al contains 13 protons and 13 electrons.

∴ magnitude of positive and negative charges in one paisa coin = $n \times z \times e$

$$\begin{aligned} &= 1.6742 \times 10^{22} \times 13 \times 1.6 \times 10^{-19} \\ &= 3.48 \times 10^4 \text{ C} \\ &= 34.8 \text{ KC} \end{aligned}$$

This is a very large amount of charge.

Hence we can conclude that ordinary neutral matter contains enormous amount of + charges.

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- ⑤ charge produces electric field and magnetic field:

A charged particle at rest produces only electric field in the space surrounding. However, if the charged particle is in unaccelerated motion it produces both electric and magnetic field. and if the motion of charged particle is accelerated it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.

- ⑥ charge resides on the surface of conductor

charge resides on the outer surface of a conductor. This is why a solid and hollow conducting sphere of same outer radius will hold maximum equal charge and a soap bubble expands on charging

(12)

Home Work:-

Q1:- calculate The charge on $^{26}\text{Fe}^{56}$ nucleus
given charge on a proton = $1.6 \times 10^{-19}\text{C}$

$$\text{Ans: } 4.2 \times 10^{-18}\text{C}$$

Q2:- calculate The charge carried by 1.25×10^8 electrons

$$\text{Ans: } -2 \times 10^{-10}\text{C}$$

Q3:- 6.25×10^{12} electrons were removed from a glass rod. What is The magnitude of positive charge acquired by The glass rod?

$$\text{Ans: } 1.0\text{MC}$$

Q4:- An electrically neutral coin of mass = 3.0g contains equal amount of positive and negative charge. Assuming The coin is made up of pure copper, what is The magnitude of The total positive (or negative) charge on The coin?

molar mass of cu = 63.5g/mol and atomic number of copper is 29

$$\text{Ans: } 1.32 \times 10^{-5}\text{C}$$

⑥

Substances

conductors

Insulators

semiconductors

conductors! - The materials which allow flow of electric charges through them easily are called conductors.

In conductors, there are large number of free electrons.

Examples! - Ag, Cu, Al, Fe, mg, Earth, bodies of human beings and animals, aqueous solutions of salts, bases and acids.
silver is the best conductor.

Insulators! - The materials which do not allow the flow of electric charges through them are called insulators.

They have negligibly small number of free electrons or no free electrons.

Ex! - glass, rubber, plastic, paraffin wax, pvc, nylon, dry wood etc

Semiconductors! - Materials having conducting properties in b/w those of conductors and insulators are called semiconductors.

Ex! - germanium and silicon are semiconductors

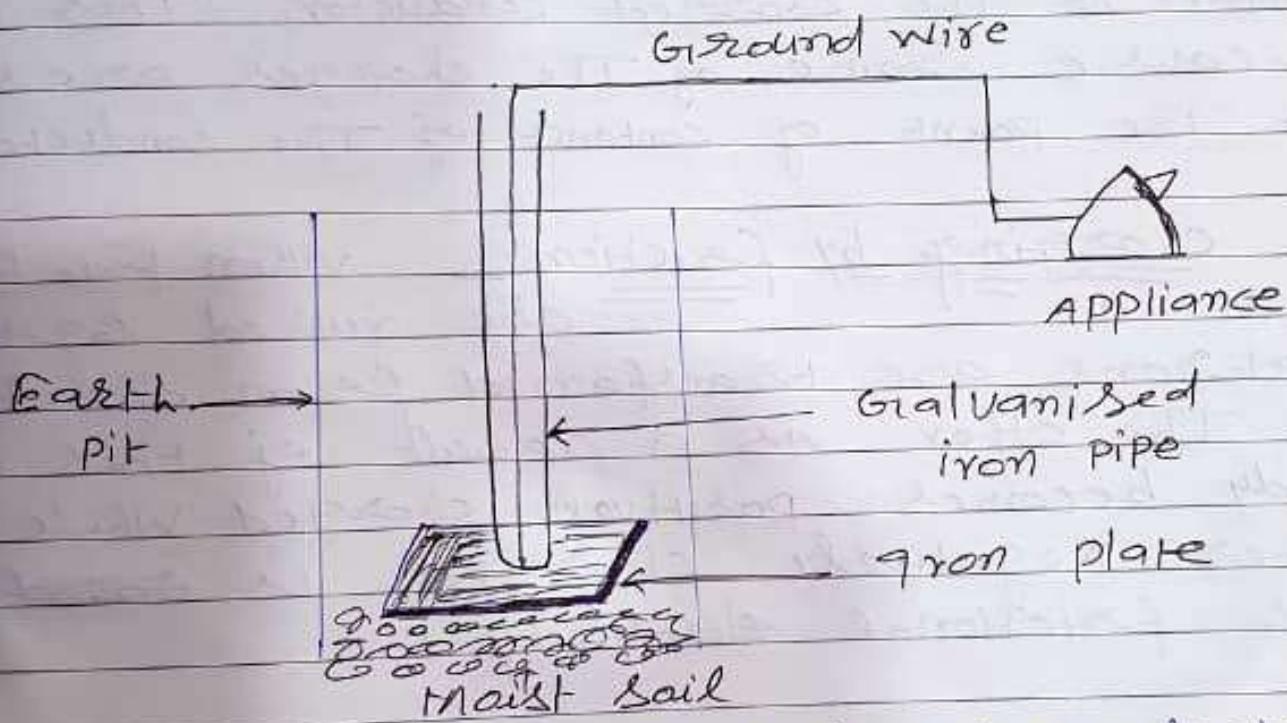
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Note:-

Dielectric:- The non-conducting material in which charges appear on its surface. When an external electric field is applied across it is known as dielectric.

Ex:- mica, impregnated paper, non-conducting oils etc.

Grounding or Earthing:- The method by which charges are shared b/w a charged body and earth is called grounding or earthing.



Grounding has great importance in electricity. It ensures safety of the appliance as well as the user. Grounding or earthing is represented by "|||".

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Electrostatic induction:- The phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farther end in the presence of ~~a~~ nearby charged body is called electrostatic induction.

Methods of charging

1:- Charging by physical contact:- If an uncharged conductor is touched with a charged conductor. The uncharged conductor may acquire charge similar to the charged conductor. This is because some of the charges are shared at the point of contact of the conductors.

2:- Charging by friction:- When two bodies are rubbed together, electrons are transferred from one body to the other. As a result of this one body becomes positively charged while the other negatively charged. is ~~called~~ known as frictional electricity.

Ex:-

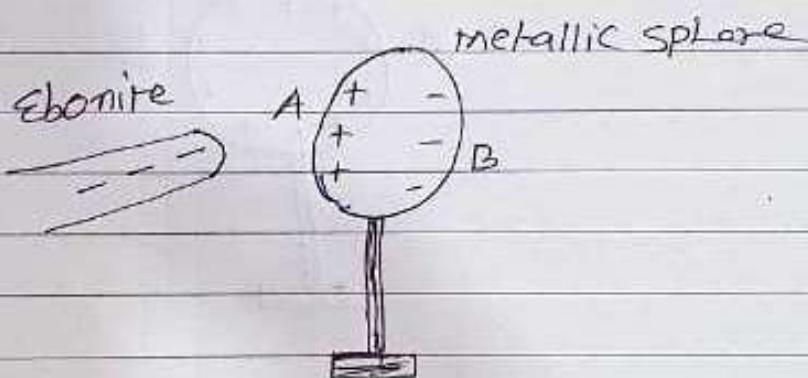
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Charging by Induction:— The process of charging a neutral conductor by placing it near a charged object is known as charging by induction.

Conceptual Questions:

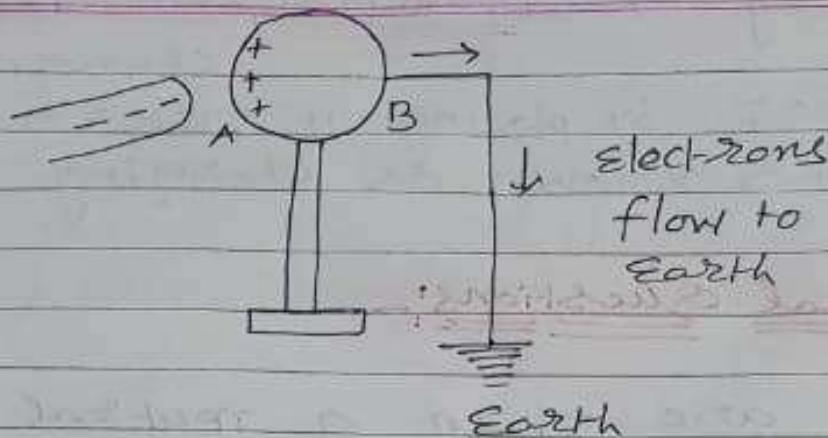
① You are given a neutral metallic sphere. How can you charge the metallic sphere with positive charge without touching it?

Ans:- ① Consider a metallic sphere placed on an insulated stand. Bring a negatively charged rod near the metal sphere without touching the sphere due to induction ~~no~~ positive charge produced at 'A' and negative charge produced at 'B'

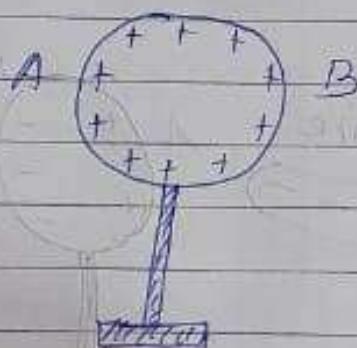


② Now connect a metallic wire with face 'B' of the sphere and the earth point. The free electrons on face 'B' of the sphere will flow to the earth through the metallic wire.

(20)



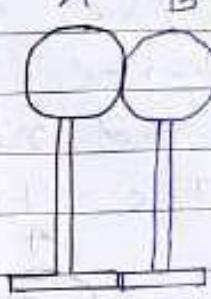
- ③ Now first remove metallic wire and then the ~~the~~ ebonite rod. Now the bound positive charge will be uniformly distributed over the surface of the sphere.
Thus the metallic sphere is positively charged.



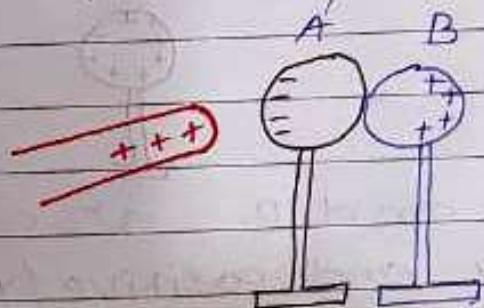
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Q21 - Consider two metallic spheres A and B, in contact, placed on insulated stands as shown in figure. What happens when

- (i) a positively charged glass rod is brought near Sphere A while taking care that the rod does ~~not~~ touch the sphere.
- (ii) Sphere A and B are ~~not~~ separated keeping the positively charged glass rod near the Sphere A.
- (iii) The glass rod is removed away from the spheres.



Ans:- (i) ~~Bring~~ Bringing positively charged glass rod near the sphere 'A' The free electrons of the spheres get attracted towards the glass rod. The farther face of Sphere B becomes short of free electrons, so it becomes positively charged.

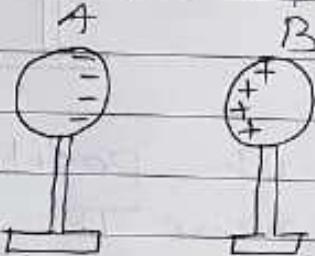


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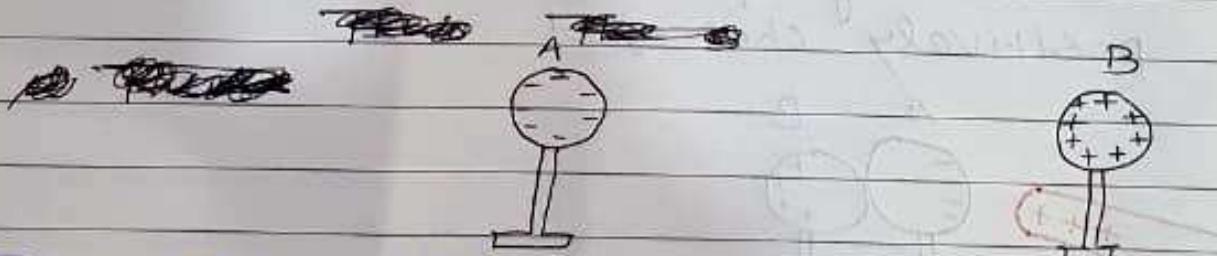
- ② Holding the glass rod near the left sphere. Separate the two spheres by a small distance, ~~now~~ The two spheres now have opposite charges,



- ③ Remove the glass rod. The charges on the spheres get redistributed. Their positive and negative charges face each other. The two spheres attract each other.



- ④ When the two spheres are separated quite apart, the charges on them get uniformly distributed

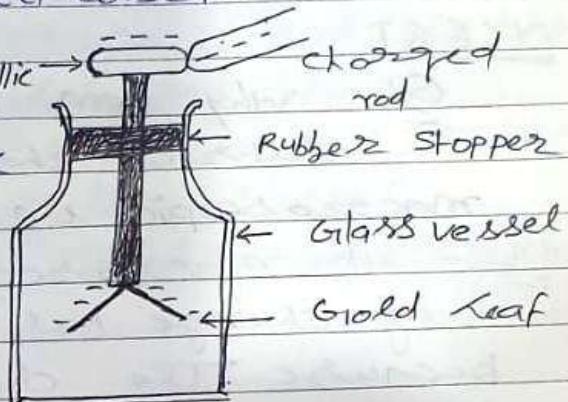


Thus the spheres A and B are charged negatively and positively respectively by the process of induction.

Gold - leaf electroscope

It is a device used for detecting an electric charge and identifying its polarity. It is known as electroscope.

It consists of a metallic disc connected with a metallic rod. A very thin foil of gold of about 5 cm length is folded in the middle and attached to the lower end of the metallic rod by cellulose tape.



Working of Gold - leaf electroscope - When a charged

object touches the metal knob at the outer end of the rod, the charge flows down to the leaves. The leaves diverge due to repulsion of the like charges they have received. The degree of divergence of the leaves gives a measure of the amount of charge.

Nature of electric charge - When a charged body is touched with an already charged electroscope, Then-

- (a) The two halves of the gold leaf converge if the charge on the charged body is of opposite sign to that of the charge on the electroscope.
- (b) The two halves of the leaf of the electroscope

(24)

diverge further if the charge on the charged charged body is of ~~some~~ some sign as that of the charge on the electroscope.

NCERT

Q1 - why can one ignore quantization of electric charge when dealing with macroscopic i.e. large scale charges?

Ans - At macroscopic level, the quantization of charge has no practical importance because the charge at macroscopic level is very large as compared to ~~electron~~ elementary charge i.e. $1.6 \times 10^{-19} C$

Ex - a small charge of 1 NC has about 10^{13} electronic charges, in such cases the charge may be treated as continuous and not quantised.

Coulomb's Law of electric force: According to this

~~law~~ "The force of attraction or repulsion b/w two stationary point charges is directly proportional to the product of the magnitudes of the two charges and inversely proportional to the square of the distance b/w them."

This force acts along the line joining the two charges.

Let r be the distance b/w two

q_1

q_2

static point charges

r

q_1 and q_2 . Then The force F of attraction or repulsion b/w them is

$$F \propto q_1 q_2 \quad \text{--- (i)}$$

$$F \propto \frac{1}{r^2} \quad \text{--- (ii)}$$

from eq. (i) and (ii) we get

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K \frac{q_1 q_2}{r^2} \quad \text{--- (iii)}$$

where K is The constant of proportionality

In C.G.S. system

$$K = 1$$

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Then from eq. (III) we get

$$F = \frac{q_1 q_2}{r^2} \text{ dyne}$$

In S.I. System

$$k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

from eq. (III) we get

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} N \quad \textcircled{IV}$$

or

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2} N \quad \textcircled{V}$$

unit charge (one coulomb charge)

$$\text{if } q_1 = q_2 = q \text{ (say)}$$

$$r = 1m$$

$$F = 9 \times 10^9 N$$

Then from eq. (V) we get

$$9 \times 10^9 N = 9 \times 10^9 \times \frac{q \times q}{1^2} N$$

$$1 = q^2$$

$$q = \pm 1$$

Hence one coulomb charge is that amount of charge that repels

an equal and similar charge with a force of $9 \times 10^9 N$ when placed in Vacuum at a distance of one metre from it.

Note:- ①

* ϵ_0 = Absolute permittivity of air or free space

$$* \quad \epsilon_0 = 8.854 \times 10^{-12} \frac{C^2}{N \cdot m^2} = \frac{\text{farad}}{m}$$

* Dimension of $\underline{\underline{\epsilon_0}}$

$$\epsilon_0 = \frac{C^2}{N \cdot m^2}$$

$$\epsilon_0 = \frac{[AT]^2}{MLT^{-2} \times L^2}$$

$$\epsilon_0 = [M^{-1}L^{-3}T^{-4}A^2]$$

② * Dimension of $\underline{\underline{\frac{1}{4\pi\epsilon_0}}}$

$$\frac{1}{4\pi\epsilon_0} = \frac{N \cdot m^2}{C^2}$$

$$\frac{1}{4\pi\epsilon_0} = \frac{MLT^{-2} \times L^2}{(AT)^2}$$

$$\frac{1}{4\pi\epsilon_0} = [M L^3 T^{-4} A^{-2}]$$

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- (III) ϵ_0 relates with absolute magnetic permeability (μ_0) and velocity of light (c) A/C to the following relation

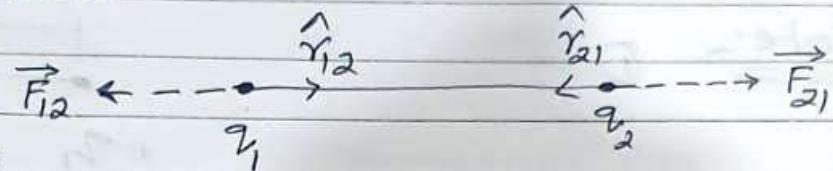
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Limitations of coulomb's law:-

- * It holds good for point charges at rest.
- * It is basically an experimental law.
- * for distances less than 10^{-15} m. it loses its validity.
- * It is applicable upto a few kilometers only.
- * It is a medium dependent law.
- * It is not a universal law.

Coulomb's Law in Vector form:-

Consider two point charges q_1 and q_2 placed in vacuum at distance r from each other.



Coulomb force acting on q_2 due to q_1 ,

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12} \quad \textcircled{1}$$

where \hat{r}_{12} is the unit vector directed from q_1 to q_2 .

Similarly, force acting on q_1 due to q_2

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad \textcircled{2}$$

But $\hat{r}_{12} = -\hat{r}_{21}$

from eq (1) we get

$$\vec{F}_{21} = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21} \quad \textcircled{3}$$

from eq (2) and (3) we get

$\vec{F}_{21} = -\vec{F}_{12}$

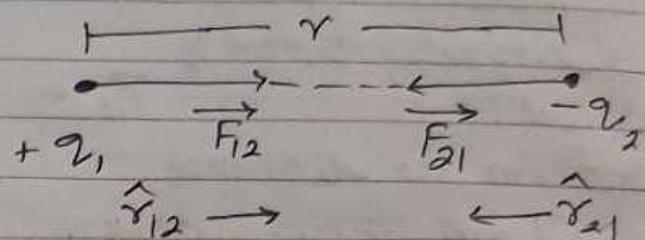
Hence The forces exerted by two point charges

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on each other are equal in magnitude and opposite in direction.

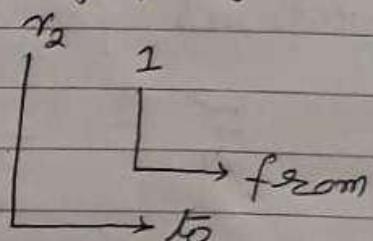
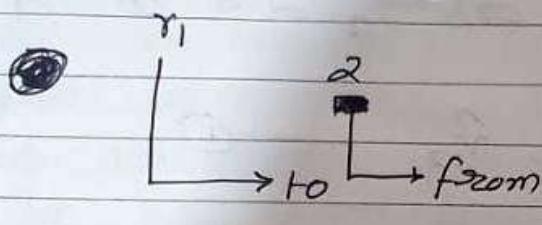
Note:

①



②

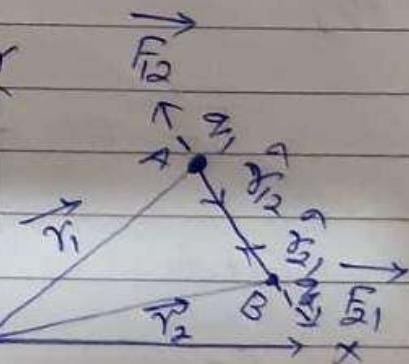
Remember convention for \hat{r}



* Coulomb's force b/w charges in terms of their position vectors:

Consider two point charges q_1 and q_2 lying in free space. Let \vec{r}_1 and \vec{r}_2 be their position vectors respectively. Coulomb's force acting on q_2 due to q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{AB}|^2} \hat{r}_{12} \quad \text{①}$$



But

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{AB} = \vec{r}_2 - \vec{r}_1 = \vec{r}_{12}$$

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from eqn (i) we get

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} \hat{r}_{12} \quad \textcircled{i}$$

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^2} \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1) \quad \textcircled{ii}$$

Similarly, coulomb's force acting on q_1 due to q_2 is

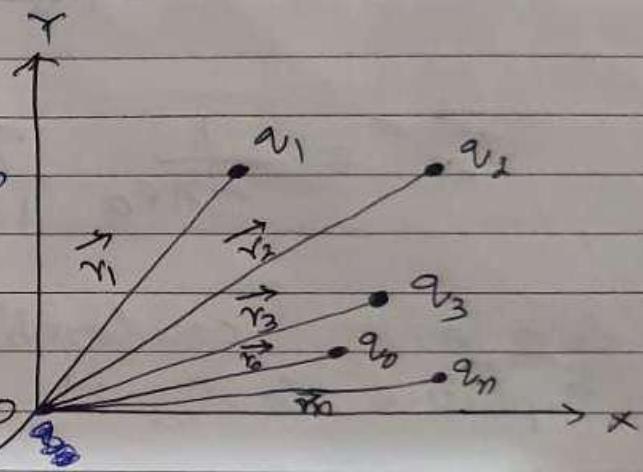
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) \quad \textcircled{iii}$$

* forces b/w multiple electric charges
 (Superposition principle)

A/c to This principle "The net force acting on a given point charge due to a number of point charges around it is the vector sum of the individual forces acting on that point charge due to all other point charges.

consider n point

charges $q_1, q_2, q_3, \dots, q_n$
 placed in vacuum at
 points whose position
 vectors w.r.t. origin
 o are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$



Let $\vec{F}_{01}, \vec{F}_{02}, \vec{F}_{03}, \dots, \vec{F}_{0n}$

be the forces F acting on the given test charge q_0 due to charges $q_1, q_2, q_3, \dots, q_n$ respectively. Then the net force acting on q_0 is

$$\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \dots + \vec{F}_{0n} \quad \text{--- (1)}$$

force acting on q_0 due to q_1 is

$$\vec{F}_{01} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_1}{|\vec{r}_0 - \vec{r}_1|^3} (\vec{r}_0 - \vec{r}_1)$$

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Force acting on q_0 due to q_2 is

$$\vec{F}_{02} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_2}{|\vec{r}_0 - \vec{r}_2|^3} (\vec{r}_0 - \vec{r}_2)$$

force acting on q_0 due to q_3 is

$$\vec{F}_{03} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_3}{|\vec{r}_0 - \vec{r}_3|^3} (\vec{r}_0 - \vec{r}_3)$$

and so on.

from ~~the~~ equation ① we get

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0 q_1}{|\vec{r}_0 - \vec{r}_1|^3} (\vec{r}_0 - \vec{r}_1) + \frac{1}{4\pi\epsilon_0} \frac{q_0 q_2}{|\vec{r}_0 - \vec{r}_2|^3} (\vec{r}_0 - \vec{r}_2) \\ + \frac{1}{4\pi\epsilon_0} \frac{q_0 q_3}{|\vec{r}_0 - \vec{r}_3|^3} (\vec{r}_0 - \vec{r}_3) + \dots + \frac{1}{4\pi\epsilon_0} \frac{q_0 q_n}{|\vec{r}_0 - \vec{r}_n|^3} (\vec{r}_0 - \vec{r}_n)$$

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \left[\sum_{i=1}^n \frac{q_i (\vec{r}_0 - \vec{r}_i)}{|\vec{r}_0 - \vec{r}_i|^3} \right]$$

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* Dielectric constant of The medium:-

force acting b/w two point charges separated by a distance r in air or vacuum

$$F_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

when The same two charges are placed same distance apart in any medium Then The force b/w them

$$F_m = \frac{1}{4\pi\epsilon_0 K} \frac{q_1 q_2}{r^2} \quad \text{--- (2)}$$

where K is dielectric ^{constant of the} medium

from eqn. (1) and (2) we get

$$\left| \frac{F_q}{F_m} = K \right| \text{ Hence The ratio of electrostatic force b/w two point}$$

$$\text{or } \left| F_m = \frac{1}{K} \cdot F_q \right| \text{ charges separated by a certain distance}$$

in air or vacuum to The electrostatic force b/w The same two charges separated by The same distance in The medium

Relative permittivity:

The ratio of the permittivity ($\epsilon_r (\epsilon_m)$) of the medium to the permittivity (ϵ_0) of free space is called relative permittivity or dielectric constant of the given medium.

i.e.

$$\epsilon_r = \frac{\epsilon_m}{\epsilon_0} = K$$

$$\therefore K = \frac{F_a}{F_m}$$

$$\Rightarrow \frac{F_a}{F_m} = \frac{\epsilon_m}{\epsilon_0}$$

Values of dielectric constant for some medium

medium	Value of K	medium	Value of K
vacuum / air	1	Rubber	6.7
water	80 or 81	wax	2
Glass	2 to 10	Metal (imp)	∞

Numericals:-

Q1.-(NCERT) What is the force b/w two small charged spheres having charges of $2 \times 10^{-7} \text{ C}$ and $3 \times 10^{-7} \text{ C}$ placed 30cm apart in air?

Ans:-

Given -

$$q_1 = 2 \times 10^{-7} \text{ C}, q_2 = 3 \times 10^{-7} \text{ C}$$

$$r = 30 \text{ cm} = 0.3 \text{ m}$$

$$F = ?$$

$$\therefore F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

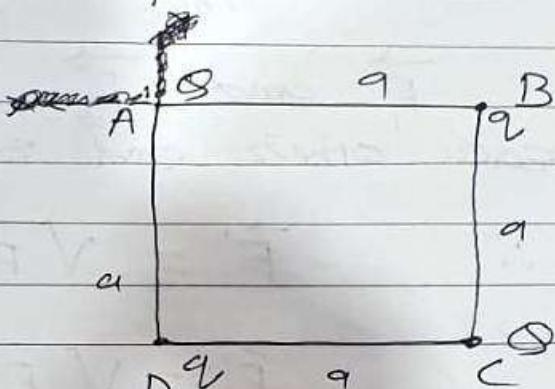
$$F = 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2}$$

$$F = \frac{9 \times 2 \times 3 \times 10^{9-7-7}}{0.3 \times 0.3}$$

$$F = \frac{9 \times 2 \times 3 \times 10^{9-14} \times 10 \times 10}{3 \times 3}$$

$$F = 6 \times 10^{-3} \text{ N} \quad (\text{repulsive})$$

Q1. Four point charges Q, q, Q and q are placed at the corners of a square of side 'a' as shown in figure. Find the resultant electric force on charge Q .



Ans:-

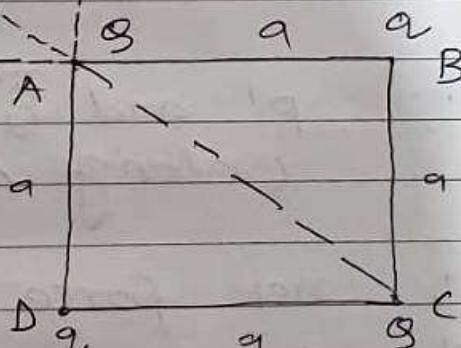
= in right angled $\triangle ADC$ by using Pythagoras

$$AC^2 = AD^2 + DC^2$$

$$AC^2 = a^2 + a^2$$

$$AC^2 = 2a^2$$

$$AC = a\sqrt{2}$$



Electric force on charge Q placed at A due to charge q placed at B

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{a^2} \text{ along } AX$$

Electric force on charge Q placed at A due to charge Q placed at C

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{Q \times Q}{(\sqrt{2})^2} = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{2a^2} \text{ along AE}$$

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Electric force on q_2 placed at A due to charge q_1 placed at D

$$\vec{F}_2 = -\frac{1}{4\pi\epsilon_0} \frac{q_2 q_1}{a^2} \text{ along } AX$$

$\therefore \vec{F}_1$ and \vec{F}_2 are perpendicular to each other and equal in magnitude

$$\therefore F' = \sqrt{F_1^2 + F_2^2 + 2FF_2 \cos 90^\circ}$$

$$\vec{F}' = \sqrt{F_1^2 + F_2^2} = \sqrt{2} \times \frac{1}{4\pi\epsilon_0} \frac{q_2 q_1}{a^2}$$

-- along AB

$\therefore F'$ and \vec{F}_2 acting in same direction
(along AB)

\therefore net force acting on charge q_2 placed at A

$$\vec{F} = \vec{F}' + \vec{F}_2$$

$$F = \sqrt{2} + \frac{1}{4\pi\epsilon_0} \frac{q_2 q_1}{a^2} + \frac{1}{4\pi\epsilon_0} \frac{q_2^2}{a^2}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_2}{a^2} \left(\sqrt{2}q_1 + \frac{q_2}{2} \right)$$

along AB

(39)

Q:- charges of 1NC , 2NC , and -3NC are placed at the corners of an equilateral triangle of side 4cm what is the magnitude of the net force acting on the charge -3NC

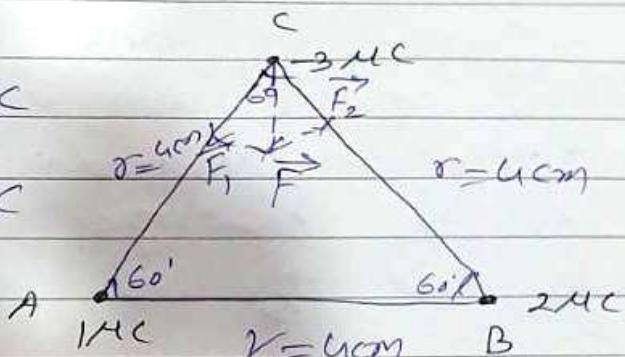
Ans: - Given -

$$q_1 = 1\text{NC} = 1 \times 10^{-6}\text{C}$$

$$q_2 = 2\text{NC} = 2 \times 10^{-6}\text{C}$$

$$q_3 = -3\text{NC} = -3 \times 10^{-6}\text{C}$$

$$r = 4\text{cm} = 4 \times 10^{-2}\text{m}$$



force acting on q_3 due to q_1 ,

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r^2}$$

$$\vec{F}_1 = 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 3 \times 10^{-6}}{(4 \times 10^{-2})^2} \text{ along CA}$$

$$\vec{F}_1 = \frac{9 \times 3 \times 10^{9-6-6+9}}{16}$$

$$\vec{F}_1 = 16.88 \text{ N along CA}$$

force acting on q_3 due to q_2

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2 q_3}{r^2}$$

(10)

$$\vec{F}_2 = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{(4 \times 10^{-2})^2}$$

$$\vec{F}_2 = 33.57 \text{ N along CB}$$

forces \vec{F}_1 and \vec{F}_2 acting at point C make an angle 60° . Then the magnitude of net force acting on the charge q_3 due to charges q_1 and q_2 is

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 60^\circ}$$

$$F = \sqrt{(16.88)^2 + (33.75)^2 + 2 \times 16.88 \times 33.75 \times \frac{1}{2}}$$

$$F = \sqrt{284.93 + 1139.05 + 269.7}$$

$$F = \sqrt{1993.68} = 44.62 \text{ N}$$

The direction of the net force F is perpendicular to the line joining A and B.

Q:- charges of $+5\text{ nC}$, $+10\text{ nC}$ and -10 nC are placed in air at the corners A, B and C of an equilateral triangle ABC having each side equal to 5cm. Determine the resultant force on the charge at A.

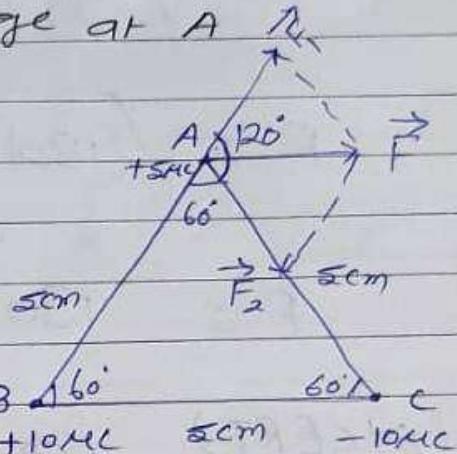
Ans:- Given -

$$q_1 = 5\text{ nC} = 5 \times 10^{-6}\text{ C}$$

$$q_2 = 10\text{ nC} = 10 \times 10^{-6}\text{ C}$$

$$q_3 = -10\text{ nC} = -10 \times 10^{-6}\text{ C}$$

$$r = 5\text{ cm} = 5 \times 10^{-2}\text{ m}$$



force at q_1 due to q_2

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\vec{F}_1 = 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 10 \times 10^{-6}}{(5 \times 10^{-2})^2} \text{ along BA}$$

$$\vec{F}_1 = 180\text{ N along BA}$$

force at q_1 due to q_3

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r^2}$$

$$\vec{F}_2 = 9 \times 10^9 \times \frac{5 \times 10^{-6} \times 10 \times 10^{-6}}{(5 \times 10^{-2})^2}$$

$$\vec{F}_2 = 180\text{ N along AC}$$

(42)

∴ The magnitude of resultant force at A

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos 120^\circ}$$

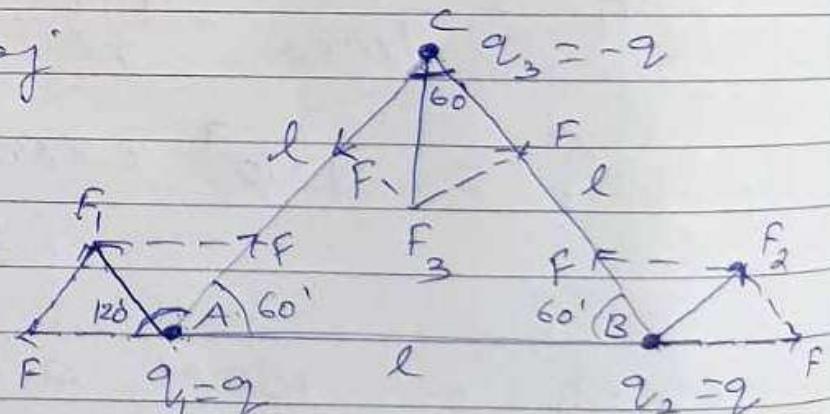
$$F = \sqrt{(180)^2 + (180)^2 + 2 \times 180 \times 180 \times \left(-\frac{1}{2}\right)}$$

$$F = 180 \text{ N along BC}$$

Q:- (NCERT) consider the charges q, q and $-q$ placed at the vertices of an equilateral triangle as shown in fig. what is the force on each charge

Ans:- magnitude of
The forces of
each charges

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{l^2}$$



The net force at q_1

$$\vec{F}_1 = \sqrt{F_1^2 + F^2 + 2FF \cos 120^\circ}$$

$$\vec{F}_1 = F \text{ along BC}$$

Similarly total force on charge q_2

(43)

$$\vec{F}_2 = F \text{ along } AC$$

Total force on charge q_3

$$\vec{F}_3 = \sqrt{F_0^2 + F^2 + 2FF \cos 60^\circ} \hat{n}$$

$$\vec{F}_3 = \sqrt{F^2 + F^2 + F^2} \hat{n} = \sqrt{3F^2} \hat{n}$$

$$\vec{F}_3 = F\sqrt{3} \hat{n}$$

where \hat{n} is a unit vector along the direction bisecting $\angle ACB$

Q1- An infinite number of charges each equal to $4\mu C$ are placed along x -axis at $x=1m, x=2m, x=4m, x=8m$ and so on find the total force on a charge of $1C$ placed at the origin.

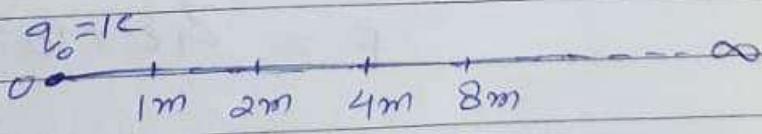
Ans:- Given -

$$q = 4\mu C = 4 \times 10^{-6} C$$

$$q_0 = 1C$$

by the principle of superposition, the total force acting on a charge on a charge of $1C$ placed at the origin

$$F = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r_1^2} + \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r_2^2} + \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r_3^2} + \dots$$



(W)

$$F = \frac{q^2 n_0}{4\pi\epsilon_0} \left[\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \dots \infty \right]$$

$$F = 9 \times 10^9 \times 1 \times 4 \times 10^{-6} \left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + \dots \infty \right]$$

$$F = 9 \times 4 \times 10^{9-6} \left[1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots \infty \right]$$

Sum of The infinite geometric progression

$$F = 36 \times 10^3 \times \frac{1}{1-\frac{1}{4}}$$

$$a = 1, r = \frac{1}{4}$$

$$F = 36 \times 10^3 \times \frac{1}{1-\frac{1}{4}}$$

$$F = 36 \times 10^3 \times \frac{4}{3}$$

$$F = 48 \times 10^3 N$$

$$F = 4.8 \times 10^4 N$$

(45)

Q1 Two point charges $+q_0$ and $+q_0$ are kept at a distance 8 m from each other. Where should we place a third charge q on the line joining the two charges so that it may be in equilibrium.

Ans:-

Let at distance

r , The third charge q be placed from $+q_0$ charge

in equilibrium condition

force b/w $+q_0$ and q = force b/w q and $+q_0$

$$\frac{1}{4\pi\epsilon_0} \frac{q_0 \times q}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q \times q}{(8-r)^2}$$

$$\frac{q}{r^2} = \frac{1}{(8-r)^2}$$

$$r^2 = q(8-r)^2$$

$$r = 3(8-r)$$

$$r = 24 - 3r$$

$$4r = 24$$

$$r = 24/4$$

$$r = 6 \text{ m}$$

(W)

Q:- A charge Q is to be divided on two objects what should be the values of the charges on the two objects so that the force b/w the objects can be maximum?

Ans:- Let q and $(Q-q)$ be the charges on the two objects Then force b/w the two objects is

$$F = \frac{1}{4\pi\epsilon_0} \frac{q(Q-q)}{r^2}$$

for F to be maximum

$$\frac{\partial F}{\partial q} = 0$$

i.e.

$$\frac{d}{dq} \left(\frac{1}{4\pi\epsilon_0} \frac{q(Q-q)}{r^2} \right) = 0$$

$$\frac{1}{4\pi\epsilon_0 r^2} \frac{d}{dq} (Qq - q^2) = 0$$

$$Q \frac{dq}{dq} - \frac{d}{dq}(q^2) = 0$$

$$Q \times 1 - 2q = 0$$

$$Q = 2q$$

$$q = Q/2$$

(A7)

$$\therefore q = (Q-q) = Q/2$$

Q1. Two charges each of $+Q$ units are placed along a line. A Third charge q is placed b/w them. At what position and for what value of q , will the system be in equilibrium?

Ans:-

Let charges Q be placed at A and B . Let charge q be placed at a distance x from charge Q at A .

So that its distance from charge Q at B is $(r-x)$.

Now, The system will be in equilibrium if force on q due to Q = force on q due to charge Q at B .

Now, The system will be in equilibrium

force on q due to Q = force on q due to charge Q at B

$$\frac{1}{4\pi\epsilon_0} \frac{q \times Q}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{q \times Q}{(r-x)^2}$$

$$\frac{1}{x^2} = \frac{1}{(r-x)^2}$$

$$x^2 = (r-x)^2$$

$$x = r - x$$

$$2x = r$$

$$x = \frac{r}{2}$$

Q8

The system will be in equilibrium if net force on any charge is zero (say placed at A)

i.e.

$$\frac{1}{4\pi\epsilon_0} \frac{Q+q}{x^2} + \frac{1}{4\pi\epsilon_0} \frac{Q+q}{r^2} = 0$$

$$\frac{1}{4\pi\epsilon_0} \frac{Q+q}{(\frac{r}{2})^2} = - \frac{1}{4\pi\epsilon_0} \frac{Q+q}{r^2} = 0$$

$$4q = -q$$

$$q = -\frac{q}{4}$$

S(H.W.) Two fixed point charges $4q$ and $2q$ are separated by a distance 'x'. Where should the third point charge q be placed for it to be in equilibrium?

(19)

Q(NCERT) Four point charges $q_A = 2\mu C$

$q_B = -5\mu C$, $q_C = 2\mu C$ and $q_D = -5\mu C$
are located at the corners

of a square ABCD of side 10cm. What
is the force on a charge $1\mu C$ placed
at the centre of the square?

Ans:- in Right angled

ΔABC by using
POT

$$AC^2 = AB^2 + BC^2$$

$$AC^2 = 10^2 + 10^2 = 200$$

$$AC = 10\sqrt{2} \text{ cm}$$

$$OA = \frac{AC}{2} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ cm} = 5\sqrt{2} \times 10^{-2} \text{ m}$$

$$\Rightarrow OA = OB = OC = OD = 5\sqrt{2} \times 10^{-2} \text{ m}$$

force on $1\mu C$ due to q_A

$$\vec{F}_A = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 1 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = \frac{18 \times 10^{9-12+4}}{25 \times 2}$$

$$\vec{F}_A = \frac{9}{25} \times 10^1 = \frac{90}{25} = 3.6 \text{ N along } OC$$

force on $1\mu C$ due to q_B

$$\vec{F}_B = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 1 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} \text{ along } OB$$

60

$$\vec{F}_B = 9 \text{ N along } OB$$

force on 1 μC due to q_c

$$\vec{F}_c = \frac{9 \times 10^9}{(2 \times 10^{-2})^2} + 2 \times 10^{-6} \times 1 \times 10^{-2}$$

$$\vec{F}_c = \frac{90}{25} \text{ N along } OA$$

$$\vec{F}_c = 3.6 \text{ N along } OA$$

force on 1 μC due to q_d

$$\vec{F}_d = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 5 \times 10^{-6}}{(5 \times 10^{-2})^2}$$

$$\vec{F}_d = 9 \text{ N along } OD$$

since \vec{F}_A and \vec{F}_c are equal in magnitude and opposite in direction. Therefore net force acting on 1 μC due to charges q_a and q_c is zero

similarly \vec{F}_B and \vec{F}_d are equal in magnitude and opposite in direction therefore net force acting on 1 μC due to charges

(51)

q_B and q_D is zero

Hence

net force acting on a charge 1nc placed at the centre of a square due to charges on the corners of the square is zero

Q(H.W.) Four charges $q_A = 3\mu C$, $q_B = -4\mu C$, $q_C = 3\mu C$ and

$q_D = -4\mu C$ are kept at the corners of a square ABCD of side 20 cm. Find the force on a charge of 1nc placed at the centre of the square.

Q. (H.W.)

Two extremely small charged copper spheres have their centres separated by a distance of 50 cm in vacuum.

(a) What is the mutual force of electrostatic repulsion of the charge on each is $6.5 \times 10^{-7} C$

(b) What will be the force of repulsion if

i) The charge on each sphere is doubled and their separation is halved?
ii) The two spheres are placed in water?